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| **Experiment No. 8** |
| **To implement All pair shortest Path Algorithm**  **(Floyd Warshall Algorithm)** |
| Date of Performance: |
| Date of Submission: |

**Experiment No. 8**

**Title:** All Pair Shortest Path

**Aim:** To study and implement All Pair Shortest Path Algorithm

**Objective:** To introduce dynamic programming-based algorithm

**Theory:** The Floyd-Warshall algorithm is a graph algorithm that is deployed to find the shortest path between all the vertices present in a weighted graph. This algorithm is different from other shortest path algorithms; to describe it simply, this algorithm uses each vertex in the graph as a pivot to check if it provides the shortest way to travel from one point to another.

Floyd-Warshall algorithm is one of the methods in All-pairs shortest path algorithms and it is solved using the Adjacency Matrix representation of graphs.

## Floyd-Warshall Algorithm

Consider a graph, **G = {V, E}** where **V** is the set of all vertices present in the graph and E is the set of all the edges in the graph. The graph, **G**, is represented in the form of an adjacency matrix, **A**, that contains all the weights of every edge connecting two vertices.

### Algorithm:

1. Construct an adjacency matrix **A** with all the costs of edges present in the graph. If there is no path between two vertices, mark the value as ∞.
2. Derive another adjacency matrix **A1** from **A** keeping the first row and first column of the original adjacency matrix intact in **A1**. And for the remaining values, say **A1[i,j]**, if **A[i,j]>A[i,k]+A[k,j]** then replace **A1[i,j]** with **A[i,k]+A[k,j]**. Otherwise, do not change the values. Here, in this step, **k = 1** (first vertex acting as pivot).
3. Repeat **Step 2** for all the vertices in the graph by changing the **k** value for every pivot vertex until the final matrix is achieved.
4. The final adjacency matrix obtained is the final solution with all the shortest paths.

**Pseudocode:**

Floyd-Warshall(w, n){ // w: weights, n: number of vertices

for i = 1 to n do // initialize, D (0) = [wij]

for j = 1 to n do{

d[i, j] = w[i, j];

}

for k = 1 to n do // Compute D (k) from D (k-1)

for i = 1 to n do

for j = 1 to n do

if (d[i, k] + d[k, j] < d[i, j]){

d[i, j] = d[i, k] + d[k, j];

}

return d[1..n, 1..n];

}

### Example:

Consider the following directed weighted graph **G = {V, E}**. Find the shortest paths between all the vertices of the graphs using the Floyd-Warshall algorithm.

A diagram of a network

Description automatically generated

**Step 1:** Construct an adjacency matrix **A** with all the distances as values.

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Description automatically generated**

**Step 2:** Considering the above adjacency matrix as the input, derive another matrix A0 by keeping only first rows and columns intact. Take **k = 1**, and replace all the other values by ***A[i,k]+A[k,j]***.

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Description automatically generated with medium confidence**

**Step 3:**

Considering the above adjacency matrix as the input, derive another matrix A0 by keeping only first rows and columns intact. Take **k = 1**, and replace all the other values by ***A[i,k]+A[k,j]***.

A number grid with numbers and equations

Description automatically generated with medium confidence

**Step 4:** Considering the above adjacency matrix as the input, derive another matrix ***A0*** by keeping only first rows and columns intact. Take **k = 1**, and replace all the other values by ***A[i,k]+A[k,j]***.

**A math problem with numbers

Description automatically generated**

**Step 5:** Considering the above adjacency matrix as the input, derive another matrix ***A0*** by keeping only first rows and columns intact. Take **k = 1**, and replace all the other values by ***A[i,k]+A[k,j]***.

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Description automatically generated with medium confidence**

**Step 6:** Considering the above adjacency matrix as the input, derive another matrix ***A0*** by keeping only first rows and columns intact. Take **k = 1**, and replace all the other values by ***A[i,k]+A[k,j]***.

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## Time Complexity Analysis:

The algorithm uses three for loops to find the shortest distance between all pairs of vertices within a graph. Therefore, the **time complexity** is **O(n3)**, where ‘n’ is the number of vertices in the graph. The **space complexity** of the algorithm is **O(n2)**.

**Program:**

**Output:**

**Conclusion:**